Name: ____________________

Reg. No.: | | | | | | | | | | |

END TERM EXAMINATION – November/December-2022/January-2023

(B.Tech. CSE/ME/CE/EEE)

Subject Code: 21AS301
Subject: Engineering Mathematics – III

Duration: 3 hours
Max. Marks: 100

Instructions

- **All Questions are compulsory**
- **The Question paper consists of 2 sections - Part A contains 10 questions of 2 marks each. Part B consists of 5 questions of 16 marks each.**
- **There is no overall choice. Only Part B question include internal choice.**

PART – A

*(2 * 10 = 20 Marks)*

1. Form partial differential equation by eliminating the arbitrary constants a, b from $z=\left(x^2+a^2\right)\left(y^2+b^2\right)$.
2. Solve the equation $(D^2-2DD'+D_1'^2)Z=0$.
3. State the Parseval's identity on Fourier transforms.
4. Find the function f(x) whose sine transform is $e^{-as}, a\geq 0$.
5. Solve $\left(D^3-2D^2D'\right)z=0$.
6. Write the one dimensional heat flow equation in steady state.
7. Find $Z\{5^k\}, k\geq 0$
8. Find the Z-transform of unit impulse function.
9. Examine the L.I. or L.D. of the set of vectors $\{(1,2,3),(3,-2,1),(1,-6,-5)\}$ in $V_3(R)$.
10. For given $T(x,y,z)=(2x-3y,7y+2z)$ and $S(x,y,z)=(x-z,y)$ write matrix associated with S + T.

PART – B

*(16 * 5 = 80 Marks)*

11. a) Find the Complex Fourier transform of $f(x)=\begin{cases}1-x^2, & |x|\leq 1\\ 0, & |x|>1\end{cases}$ hence prove that $\int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos\left(\frac{s}{2}\right)ds=\frac{3\pi}{16}$ and $\int_0^\infty\left[\frac{\sin s - s\cos s}{s^3}\right]^2 ds=\frac{\pi}{15}$.

OR

b) Find the Fourier transform of $f(x)=\begin{cases}1, & \text{if } |x|\leq 1\\ 0, & \text{if } |x|>1\end{cases}$

And hence show that $\int_0^\infty \frac{\sin\theta}{\theta}d\theta=\frac{\pi}{2}$ and $\int_0^\infty \frac{\sin^2 x}{x}dx=\frac{\pi}{2}$.

12.(i) Solve $(D^3 - 7DD'^2 - 6D'^3)z = x^2 y + \sin(x+2y)$.

(ii) Solve $\left(D^2 - DD' + D'^2\right)z = 2x + 3y$.

OR

(i) Solve $(D^2 - 2DD')z = x^3 y + e^{2x}$

(ii) Solve $(D^3 - 7DD'^2 - 6D'^3)z = e^{2x+y}$

13a) A uniform elastic string of length 60 cms is stretched and fastened at the ends and the initial displacement is $60x - x^2$ while the initial velocity is zero. Find the displacement function $y(x, t)$.

OR

b) Solve the boundary value problem $\dfrac{\partial u}{\partial t} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}$ subject to the conditions

$$u(0, t) = 0 \quad for\ t \geq 0,$$
$$u(l, t) = 0 \quad for\ t \geq 0$$
$$u(x, 0) = \sin\left(\frac{\pi x}{l}\right).$$

14.(i) Solve the difference equation by using Z-Transforms

$$6y_{k+2} - y_{k+1} - y_k = 0, y(0) = 0, y(1) = 0$$

(ii) Find the Z-Transforms of $\left\{\left(\frac{1}{2}\right)^{|k|}\right\}$

OR

Find the inverse Z-Transform of $\frac{1}{(z-a)(z-b)}$; $a < b$

(i) $|z| < a$ (ii) $a < |z| < b$ (iii) $|z| > b$.

15. (i) Show that the function $T: R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (x_1, x_2, 0)$ is a linear transformation.

(ii) Show that the function $T: R^2 \to R^3$ defined by $T(x, y) = (x, x - y, x + y)$ is a linear transformation and is One to one but not onto.

OR

Show that the set V of all real valued continuous functions of x defined on [0,1] is a vector space over the field R of real numbers w.r.t pointwise vector addition and scalar multiplication defined by:

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) \forall f_1, f_2 \in V;$$
$$(af_1)(x) = af_1(x) \forall a \in R, f_1 \in V.$$

$m^2 = m + 1$

$z^2 - 2z + z - 1$

$z(z-2) + z(1 \cdot 2)$